

# My Favourite Open Problem in Database Theory

(and a few other things)

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## **Part One: the Few Other Things**

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# Overview

# Computer Science

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(theoretical) **Computer Science**

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Problem Solving

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(finding creative explanations)

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(theoretical)

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## Theory Building

(finding creative explanations)

## Problem Solving

(finding interesting solutions)

## Decision Problems

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# Entscheidungsproblem

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David Hilbert

the father of mathematical logic



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Decision problem:

Does there exist an algorithm that  
considers an inputted statement and  
answers "yes" or "no" according to  
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Halting problem:

Can one write a program deciding whether the input's program execution halts?

# Programming in strange languages

## Hilbert's 10th problem

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If we could have such a program, then we would be able to make an algorithm deciding halting problem!

## Part Two: The Problem

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# My favourite database theory problem

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Can one *decide* query containment problem?

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## Query Containment Problem

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Can one *decide* query containment problem?

What kind of query? What is a database in this context?

# Queries

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$$P = V \vee \exists x, z M(x, z)$$

# Boolean queries and homomorphisms

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$$\text{Vee} = \exists x, y, z \ E(x, y) \wedge E(x, z) \quad \text{Tri} = \exists x, y, z \ E(x, y) \wedge E(y, z) \wedge E(z, x)$$

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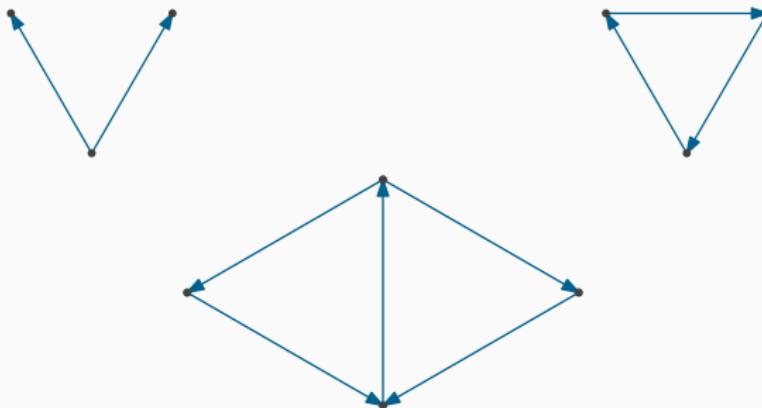
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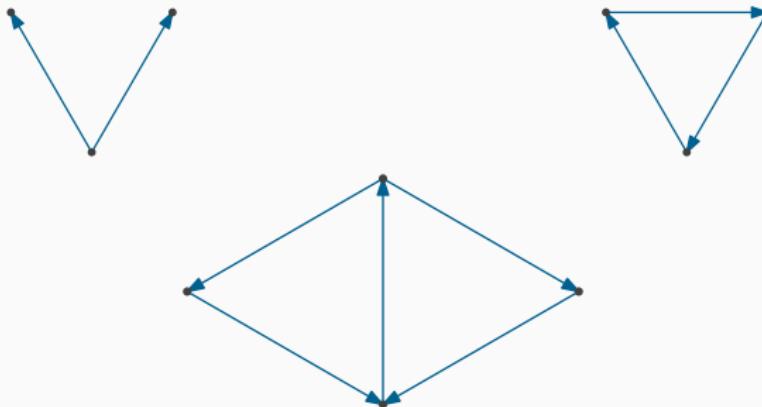
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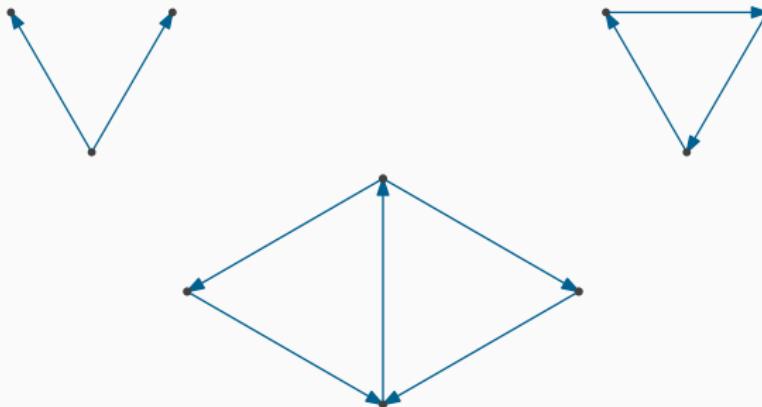
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set semantics	
bag semantics	

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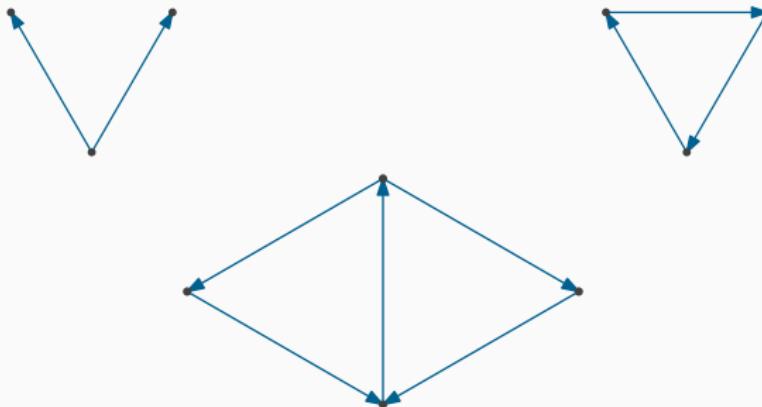
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	$\text{Vee}(\mathcal{D})$	$\text{Tri}(\mathcal{D})$
set semantics		
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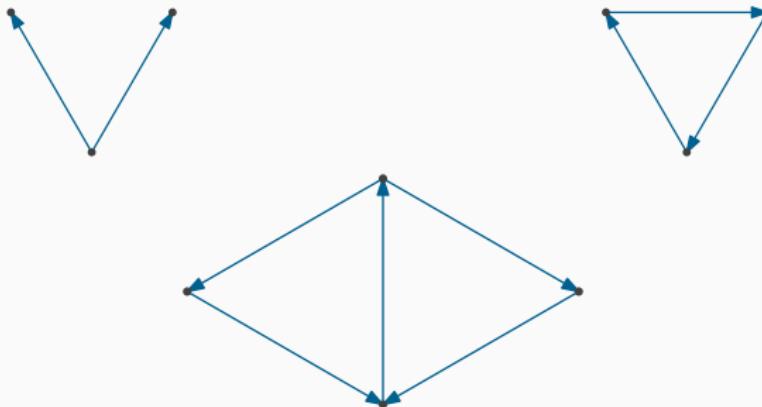
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$Q(\mathcal{D})$	$\text{Vee}(\mathcal{D})$	$\text{Tri}(\mathcal{D})$
set semantics	yes	yes
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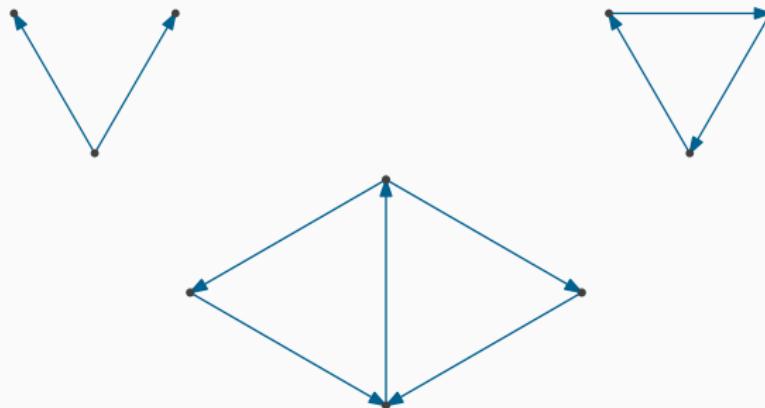
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set semantics	yes	yes
bag semantics	7	6

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**CQ Containment:** Still **open** after 30 years!

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**UCQ Containment:** Closed and **undecidable**

# UCQ containment under bag semantics

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$$Q_s = \exists z \ \textcolor{blue}{X}(z) \wedge \textcolor{blue}{X}(z) \ \vee \ \textcolor{red}{Y}(z) \wedge \textcolor{red}{Y}(z) \wedge \textcolor{red}{Y}(z)$$

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$\mathcal{D}_2$		

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$$\mathcal{D}_1 \quad \begin{matrix} \textcolor{blue}{X} & \textcolor{blue}{X} & \textcolor{blue}{X} & \textcolor{red}{Y} \\ \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$\mathcal{D}_2 \quad \begin{matrix} \textcolor{blue}{X} & \textcolor{blue}{X} & \textcolor{red}{Y} & \textcolor{red}{Y} \\ \cdot & \cdot & \cdot & \cdot \end{matrix}$$

---

$Q(\mathcal{D})$	$Q_b$	$Q_s$
$\mathcal{D}_1$	$\textcolor{blue}{3}^2 + \textcolor{red}{1}^3$	$2 \cdot \textcolor{blue}{3} \cdot \textcolor{red}{1}$
$\mathcal{D}_2$	$\textcolor{blue}{2}^2 + \textcolor{red}{2}^3$	$2 \cdot \textcolor{blue}{2} \cdot \textcolor{red}{2}$

---

$$Q_s(\mathcal{D}) \subseteq Q_b(\mathcal{D}) \iff 2\textcolor{blue}{x}\textcolor{red}{y} \leq \textcolor{blue}{x}^2 + \textcolor{red}{y}^3$$

## UCQ containment under bag semantics

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---

$Q(\mathcal{D})$	$Q_b$	$Q_s$
$\mathcal{D}_1$	$3^2 + 1^3$	$2 \cdot 3 \cdot 1$
$\mathcal{D}_2$	$2^2 + 2^3$	$2 \cdot 2 \cdot 2$

---

$$Q_s(\mathcal{D}) \subseteq Q_b(\mathcal{D}) \iff 2\textcolor{blue}{x}\textcolor{red}{y} \leq \textcolor{blue}{x}^2 + \textcolor{red}{y}^3$$

Reduction from a variant of Hilbert's 10th problem!

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*Bag Semantics Query Containment: The CQ vs. UCQ Case and Other Stories*  
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Thank you!