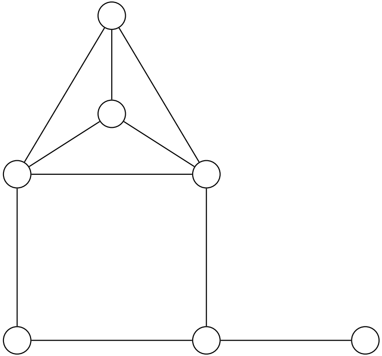
Algorithmics of Dynamic Well-Structured Graphs

Marek Sokołowski

16 October 2025

Graphs & Graph problems



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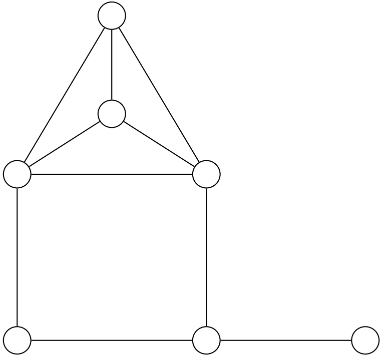

n vertices, m edges

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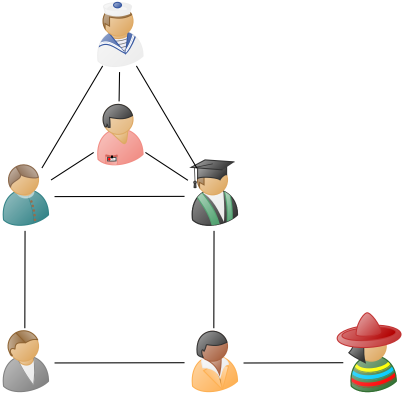


n vertices, m edges

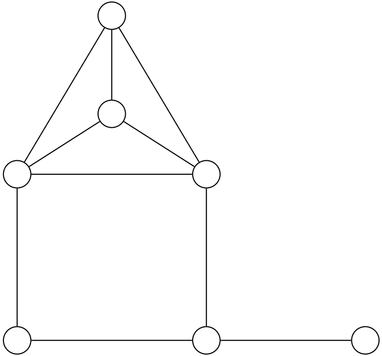
people **relationships**

Graphs & Graph problems

n vertices, m edges

intersections **streets**

Graphs & Graph problems



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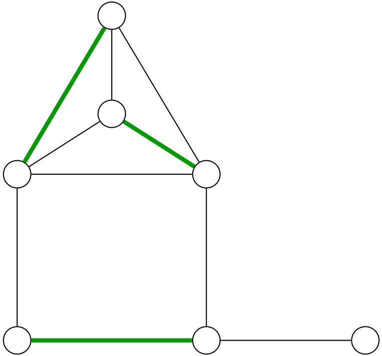

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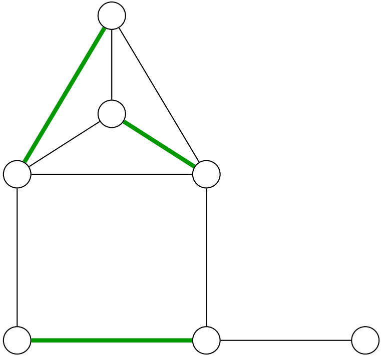
MAXIMUM MATCHING

Graphs & Graph problems



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[Edmonds '61]

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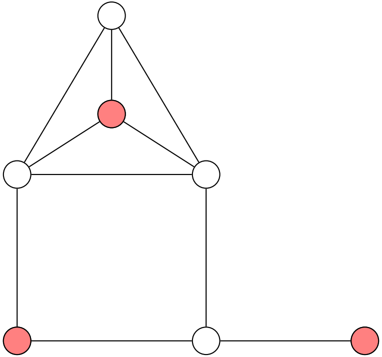

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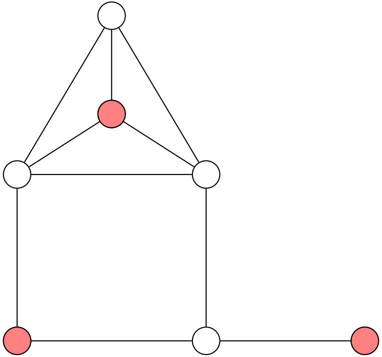
MAXIMUM INDEPENDENT SET

Graphs & Graph problems



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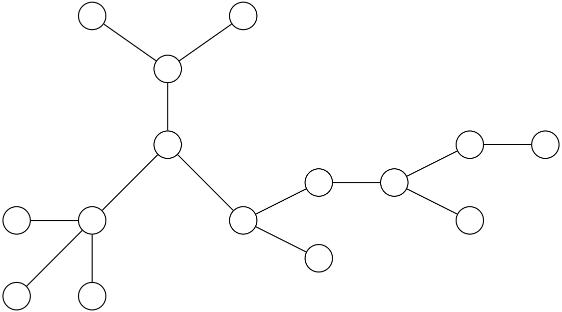
Maximum Independent Set

NP-hard!

[Cook '71, Karp '72, Levin '73]

Trees

Maximum Independent Set is NP-hard in general... But becomes easy on **trees**!



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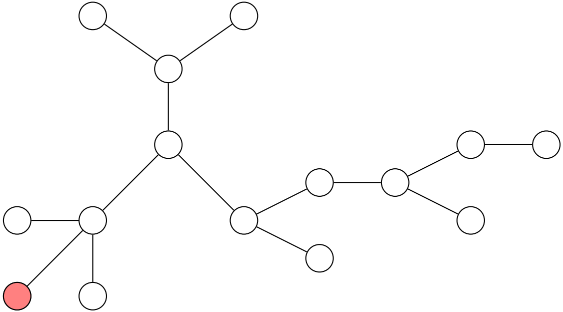

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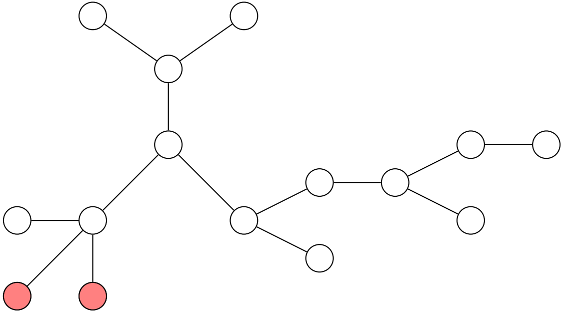

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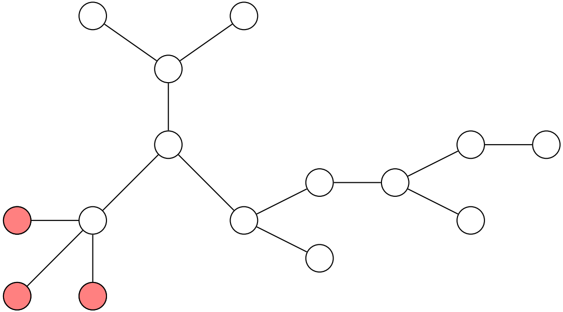

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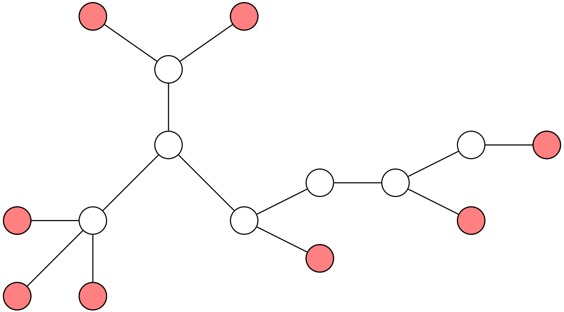

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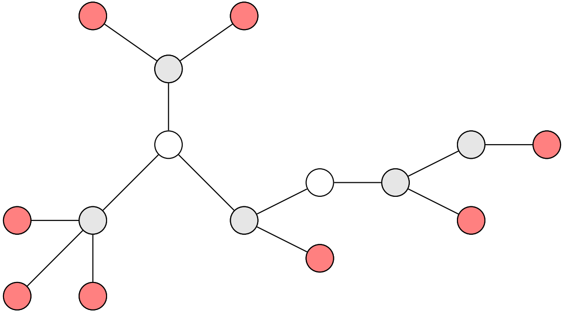

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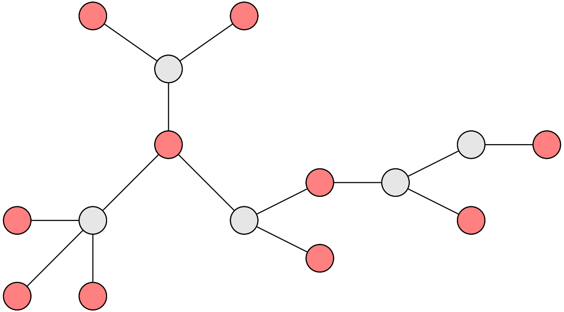

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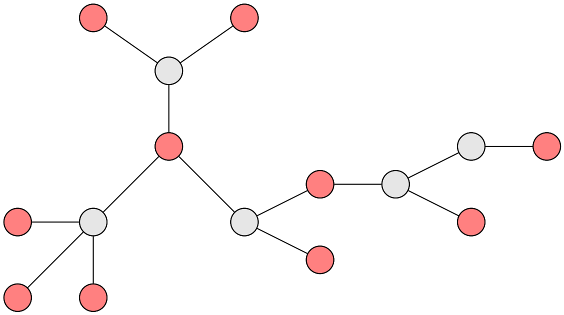
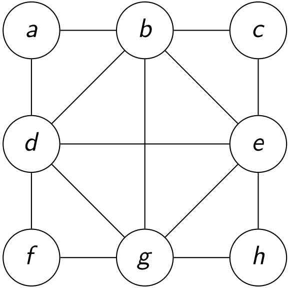


Question

Maybe some hard problems can be solved efficiently on **more general tree-like** graphs?

Treewidth



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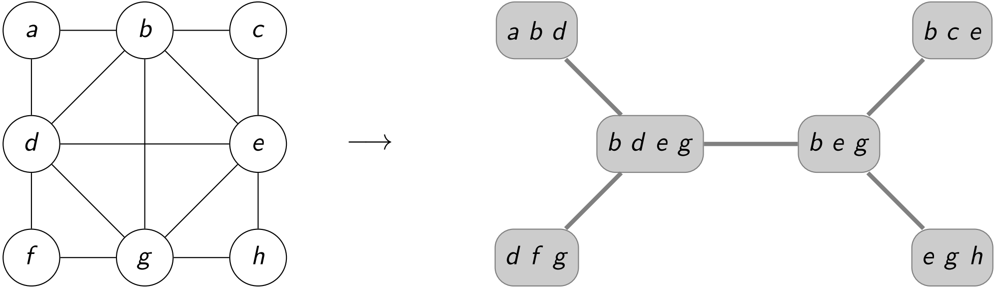
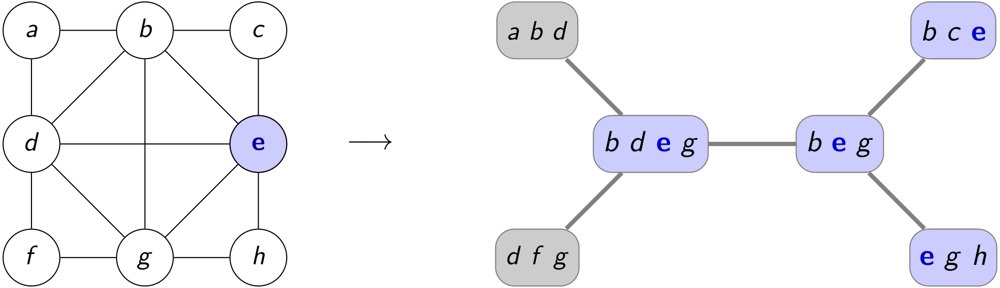


tree decomposition

Treewidth



- Each **vertex** in a non-empty connected subgraph of the decomposition

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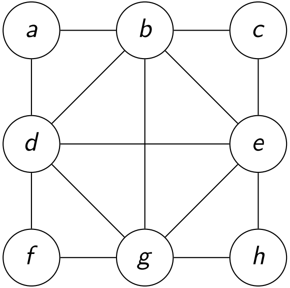
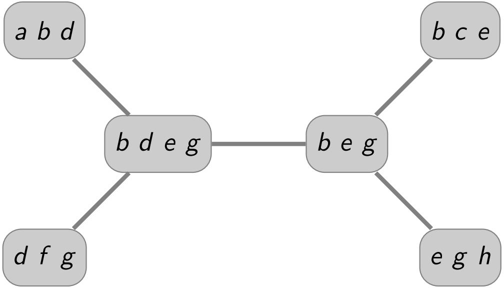


$\longrightarrow$



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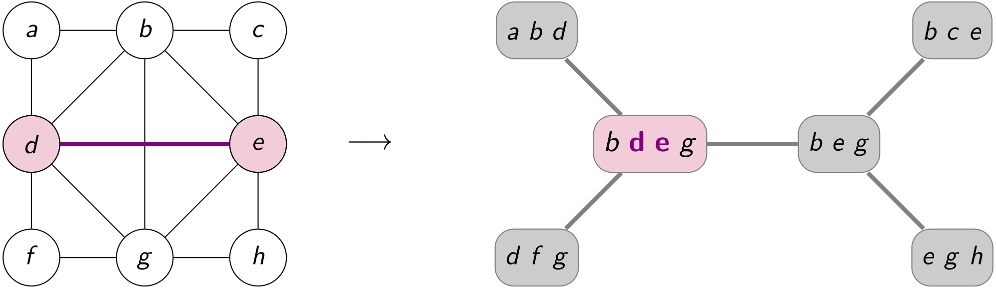

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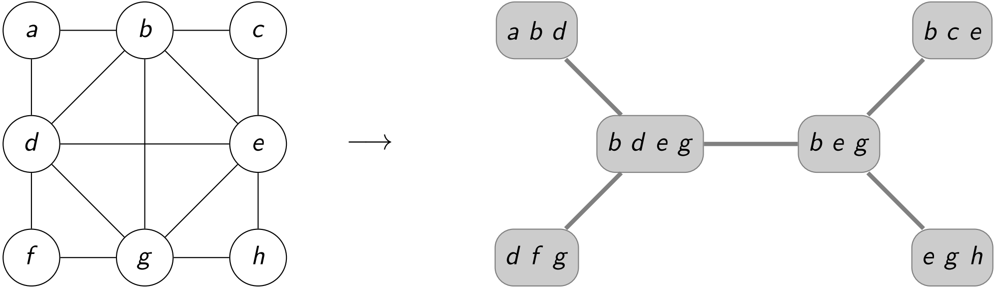

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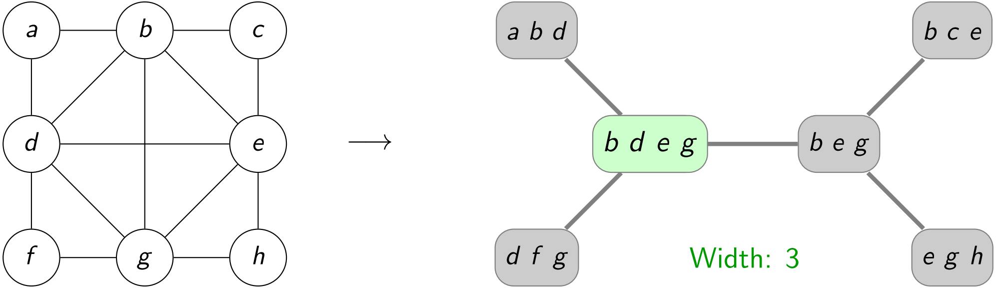

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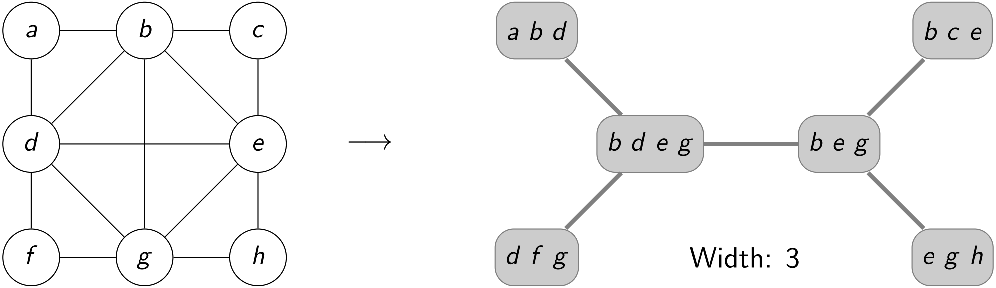

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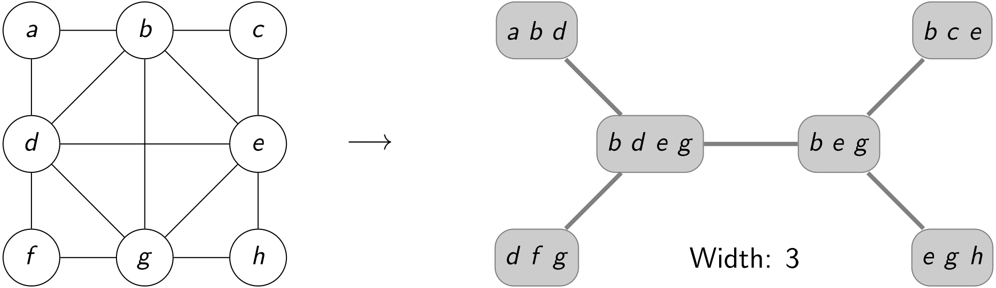

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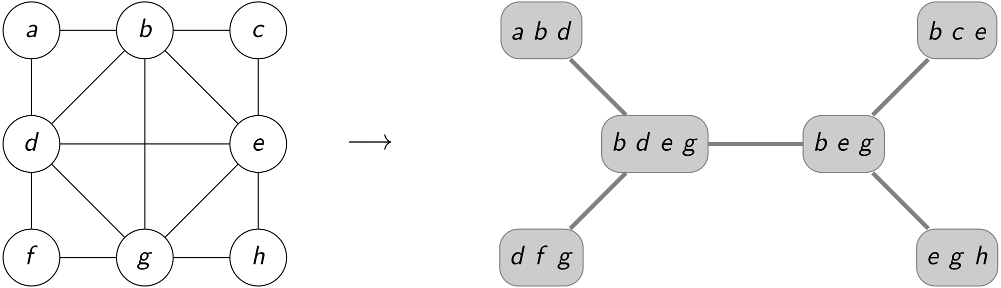
- **Treewidth:** minimum possible width of a tree decomposition

Treewidth



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Given: n-vertex graph G and its tree decomposition of width w

Then: Maximum Independent Set can be solved in time $2^{\mathcal{O}(w)} \cdot n$

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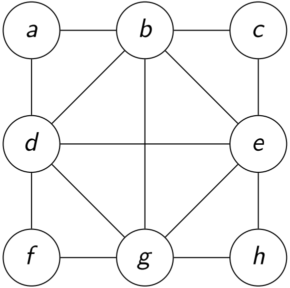

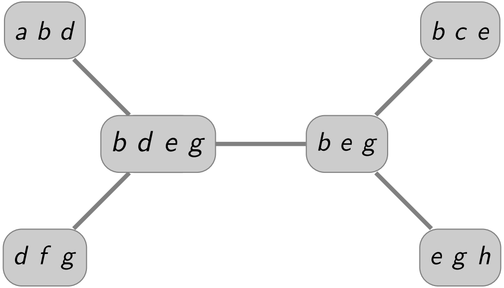

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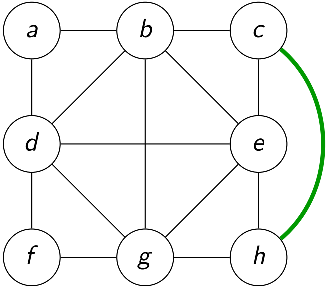

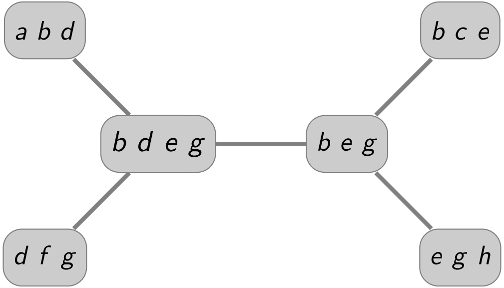

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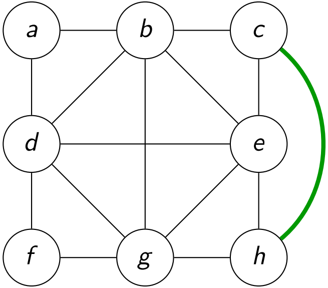

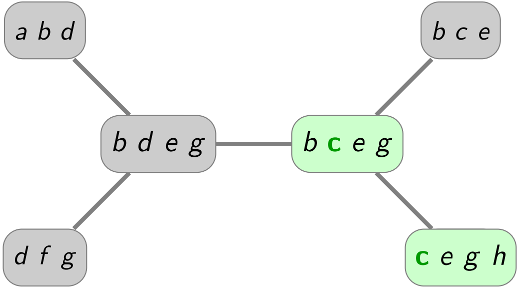

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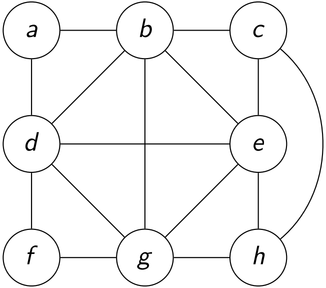

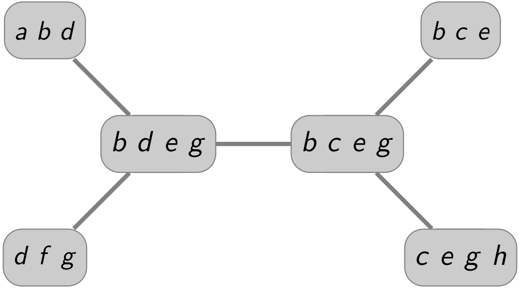

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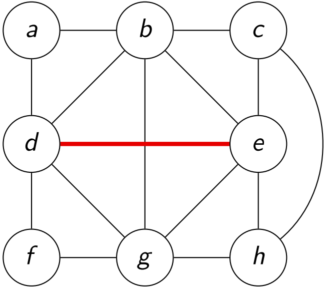

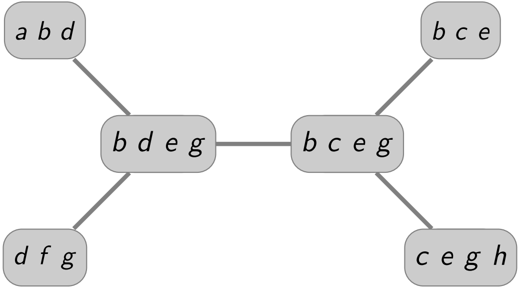

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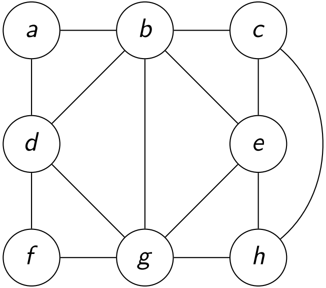

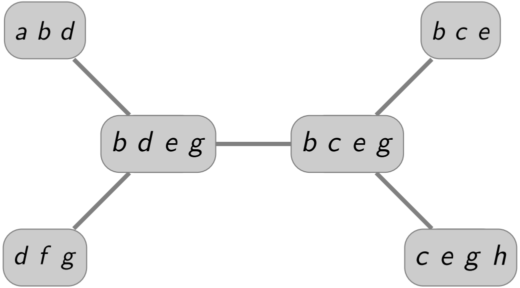

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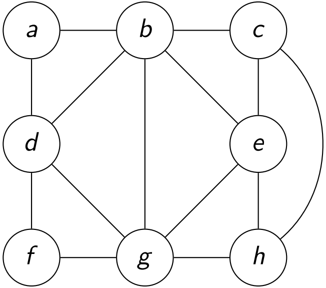

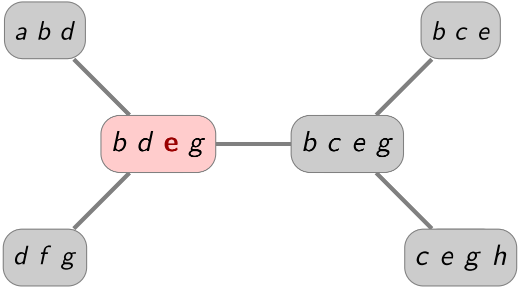

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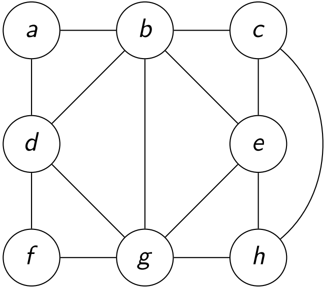

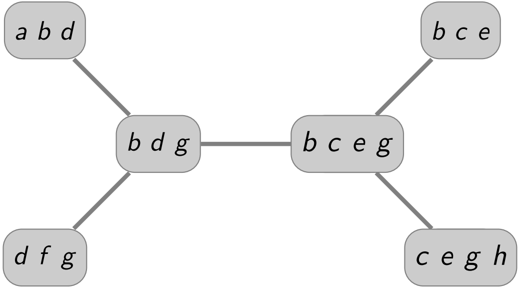

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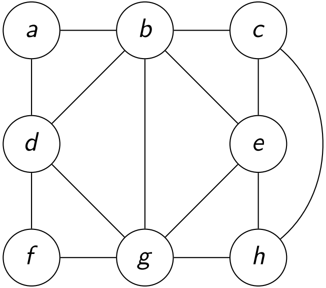

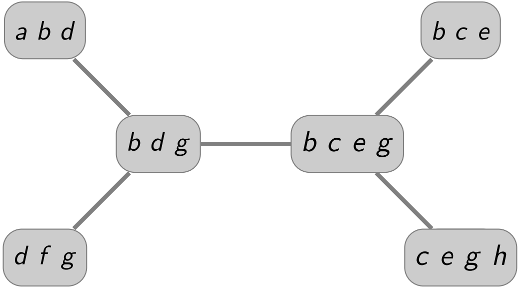

Problem

How to maintain tree decompositions of **dynamic graphs**?

Dynamic Treewidth

Korhonen, Majewski, Nadara, Pilipczuk, **Sokołowski** [FOCS '23]

Dynamic Treewidth

Main result

Dynamic Treewidth

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DYNAMIC TREEWIDTH

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In a **dynamic graph** G with n vertices of treewidth $w \dots$

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In a **dynamic graph** G with n vertices of treewidth w ...

We maintain: a tree decomposition of G of width at most $6w + 5$ ...

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$$\log^{1000} n \quad \ll \quad \mathbf{2}^{\sqrt{\log \mathbf{n} \log \log \mathbf{n}}} \quad \ll \quad n^{0.001}$$

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Extension

We can also dynamically solve any decision/optimization problem expressible in CMSO_2 logic.

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MAX MATCHING, MAX INDEPENDENT SET, LONGEST PATH, HAMILTONIAN CYCLE...

Dynamic Treewidth: Follow-Up

Korhonen [FOCS '25]

DYNAMIC TREEWIDTH IN LOGARITHMIC TIME

Follow-up result

In a **dynamic graph** G with n vertices of treewidth $w\ldots$

We maintain: a tree decomposition of G of width at most $9w + 8\ldots$

Initialization time: $2^{\mathcal{O}(w)} \cdot n$

Update time: $2^{\mathcal{O}(w)} \cdot \log n$ *(amortized)*

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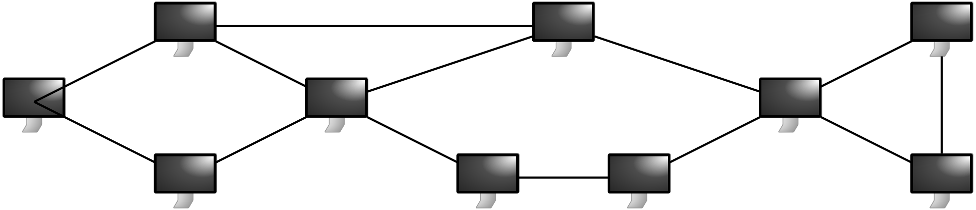
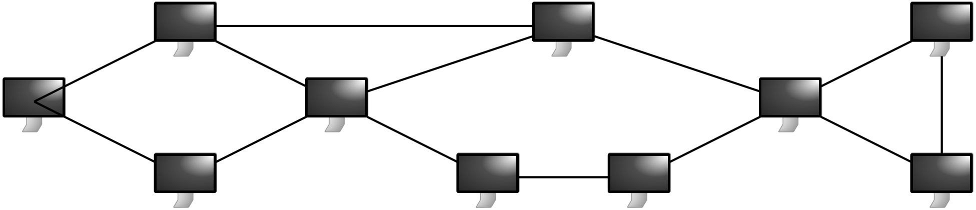
MAX MATCHING, MAX INDEPENDENT SET, LONGEST PATH, HAMILTONIAN CYCLE...

Biconnectivity



Biconnectivity



x, y **biconnected** $\iff$ in the same connected component, not separated by another vertex

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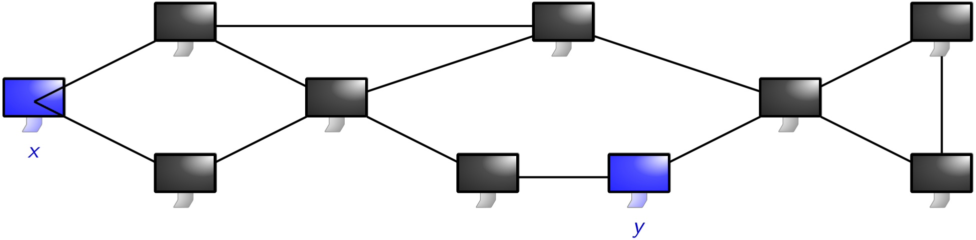

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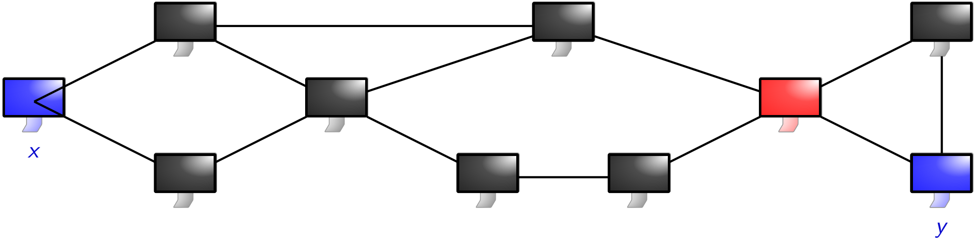

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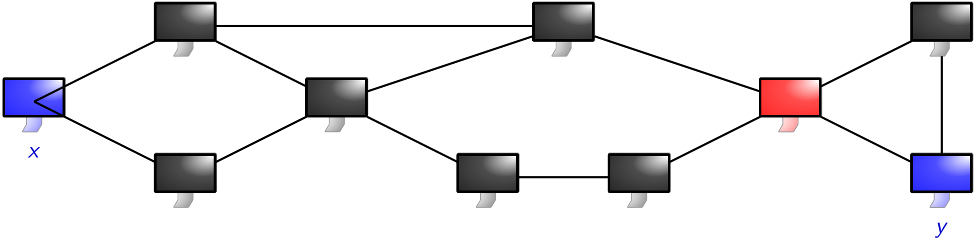
Dynamic Biconnectivity



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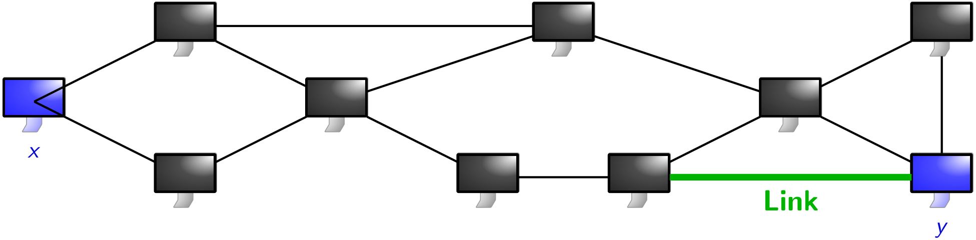

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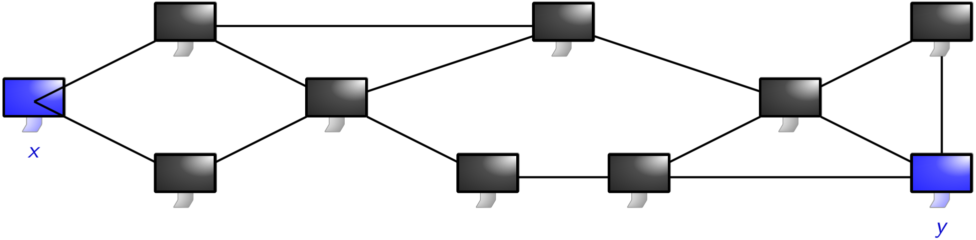

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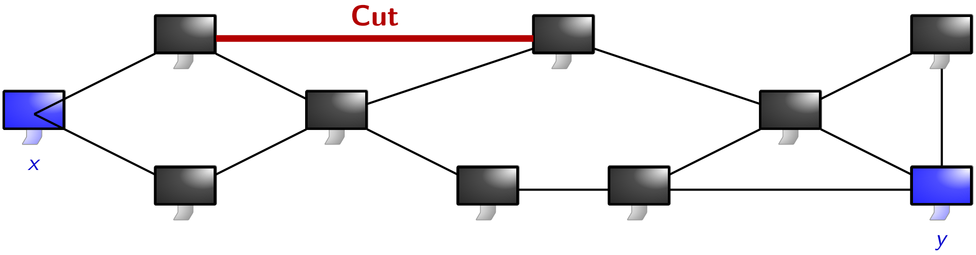

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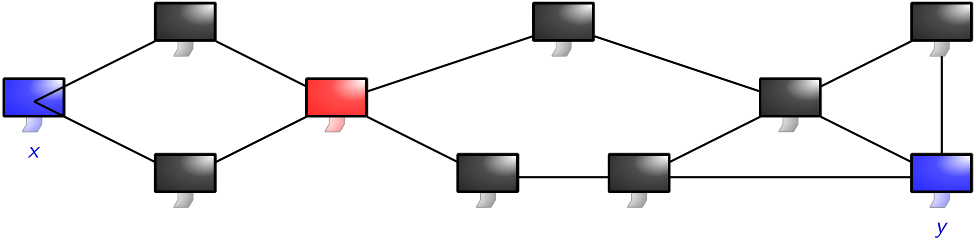

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Dynamic Biconnectivity

Holm, Nadara, Rotenberg, **Sokołowski** [STOC '25]

FULLY DYNAMIC BICONNECTIVITY IN $\widetilde{O}(\log^2 n)$ TIME

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FULLY DYNAMIC BICONNECTIVITY IN $\widetilde{O}(\log^2 n)$ TIME

	Update/Query Time	**Deterministic?**
[Henzinger '92]	$\mathcal{O}(m^{2/3})$	yes

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[Henzinger, King '95]	$\mathcal{O}(\log^4 n)$	**no**

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[Thorup '00]	$\mathcal{O}(\log^4 n \log \log n)$	yes
our work	$\mathcal{O}(\log^2 n \log^2 \log n)$	yes

Thank you!

Extra slides: Dynamic Rankwidth

Treewidth, but for denser graphs

Issue: treewidth applicable only to **sparse** graphs. . .

But there also exist **dense** tree-like graphs!

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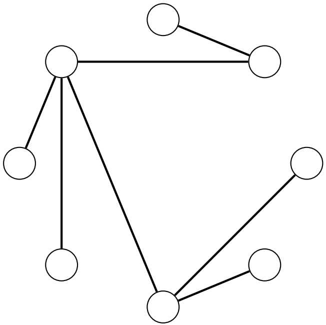

trees

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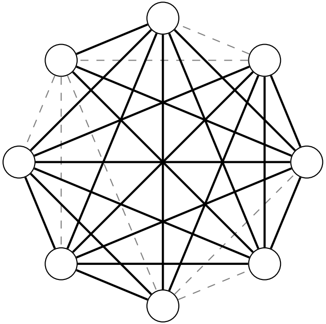

complements of trees

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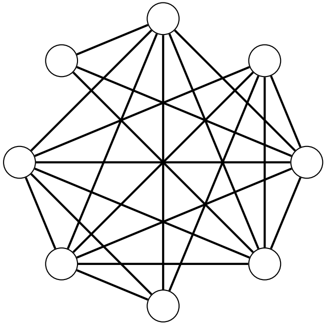

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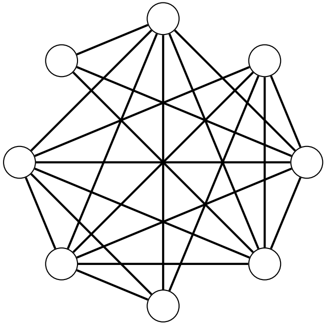

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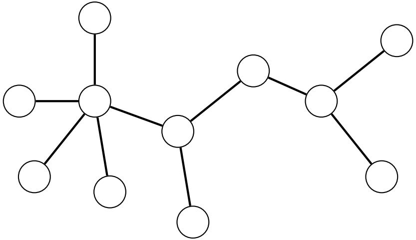

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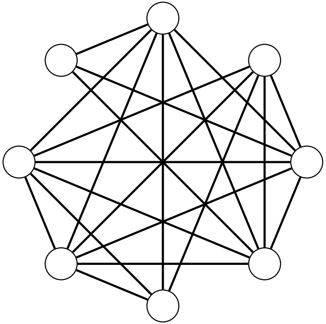
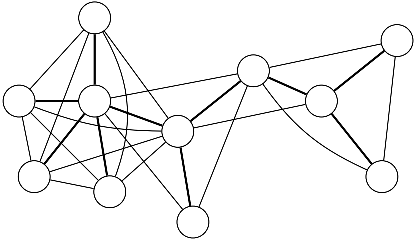


complements of trees



squares of trees

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Solution

Equivalent notions of **cliquewidth** [Courcelle et al. '93] and **rankwidth** [Oum, Seymour '06].

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Rankwidth is great!

Given: n-vertex graph G and its **rank decomposition** of width w
Then: Maximum Independent Set can be solved in time $2^{f(w)} \cdot n$

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Also Max Clique, Min Dominating Set, Longest Induced Path, ...

Rankwidth

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Same problem: Need to compute a rank decomposition.

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Same problem: Need to compute a rank decomposition.

Rank decomposition algorithms

Given an n-vertex graph G of rankwidth w, we can **find** a rank decomposition of G...

	Width guarantee	**Time**
[Oum, Seymour '06]	$3w + 1$	$2^{\mathcal{O}(w)} \cdot n^9$
[Oum '08]	$3w - 1$	$f(w) \cdot n^3$
[Jeong, Kim, Oum '21]	w	$f(w) \cdot n^3$
[Fomin, Korhonen '22]	w	$f(w) \cdot n^2$

Dynamic Rankwidth

Korhonen, **Sokołowski** [STOC '24]

ALMOST-LINEAR TIME PARAMETERIZED ALGORITHM FOR RANKWIDTH VIA DYNAMIC RANKWIDTH

Main result

Dynamic Rankwidth

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In a **dynamic graph** G with n vertices and m edges of rankwidth w ...

Dynamic Rankwidth

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We maintain: a rank decomposition of G of width at most $4w$ ...

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Extension

We can also dynamically solve any decision/optimization problem expressible in CMSO_1 logic.

Dynamic Rankwidth

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Dynamic Rankwidth

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[Korhonen, Sokołowski '24]	w	$f(w) \cdot n^{1+o(1)} + \mathcal{O}(m)$